

FIGURE 15.3 1-kHz sine wave sampled at 10 kHz.

spaced sampling intervals are shown at an arbitrary phase offset from the sine wave to illustrate the arbitrary alignment of samples to the incoming signal.

Discrete digital samples are reconstructed to generate an analog output that is a facsimile of the original analog input minus a finite degree of distortion. Figure 15.4 shows the 1-kHz sine wave samples as they are emitted from a DAC. On its own, the sampling process creates a stair-step output that bears similarity to the original signal but is substantially distorted. The output is not a pure sine wave and contains added high-frequency components at the sampling frequency—10 kHz in this case. This undesirable stair-step output must be passed through a lowpass filter to more closely reconstruct the original signal. A lowpass filter converts the sharp, high-frequency edges of the DAC

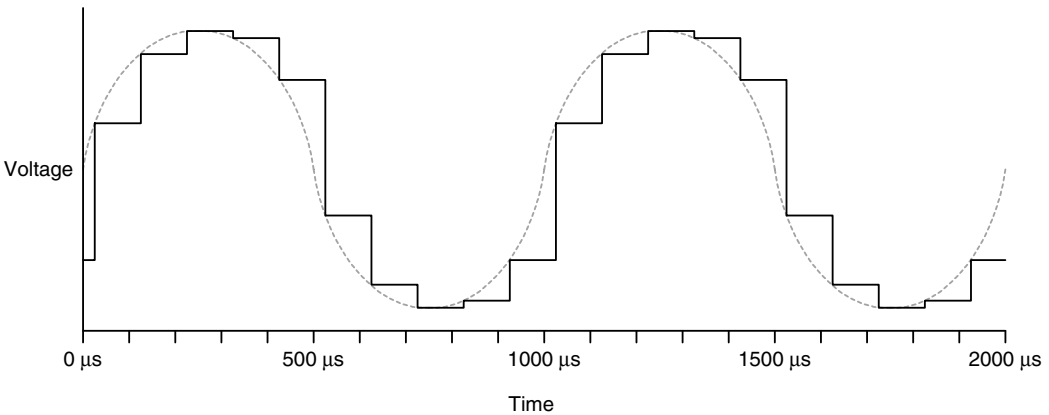


FIGURE 15.4 Reconstructed 1-kHz sine wave without filtering.

output into smoother transitions that approximate the original signal. The output will never be identical to the original signal, but a combination of sufficient sampling rate and proper filter design can come very close.

As the ratio of sampling rate to signal frequency decreases, the conversion accuracy becomes more coarse. Figure 15.5 shows the same 1-kHz sine wave being sampled at only three times the signal frequency, or 3 kHz. Consider how the stair-step output of a DAC looks with this sampling scheme. Here, a filter with sharper roll-off is required, because the undesirable frequency components of the DAC output are spaced only 2 kHz apart from the frequency of interest instead of nearly a decade as in the previous example. A suitable filter would be able to output a nearly pure 1-kHz sine wave, but the signal's amplitude would not match that of the input, because the maximum DAC output voltage would be less than the peak value of the input signal, as indicated by the position of the samples as shown.

Preservation of a signal's amplitude is not critical, because a signal can always be amplified. It is critical, however, to preserve the frequency components of a signal because, once lost, they can never be recovered. Basic sampling theory was pioneered by Harry Nyquist, a twentieth century mathematician who worked for Bell Labs. Nyquist's theorem states that a signal must be sampled at greater than twice its highest frequency to enable the preservation of its informational, or frequency, content. Nyquist's theorem assumes a uniform sampling interval, which is the manner in which most data conversion mechanisms operate. The *Nyquist frequency* is a common term that refers to one-half of the sampling frequency. A data conversion system is said to operate below the Nyquist frequency when the applied signal's highest frequency is less than half the sampling frequency.

Nyquist's theorem can be understood from a qualitative perspective by considering operation at exactly the Nyquist frequency. Figure 15.6 shows a 1-kHz sine wave being sampled at 2 kHz. There are a range of possible phase differences between the sampling interval and the signal itself. The worst-case alignment is shown wherein the samples coincide with the sine wave's zero crossings. Because each sample is identical, the observed result is simply a DC voltage.

If the sampling frequency is increased by a small amount so that the signal is less than the Nyquist frequency, it is impossible for consecutive samples to align themselves at the signal's zero crossings. The amplitude measurement may suffer substantially, but the basic information content of the signal—that it is a 1-kHz sine wave—will not be lost. As the prior examples show, operating with substantially higher sampling rates, or operating at substantially below the Nyquist frequency, increases the accuracy of the data conversion process.

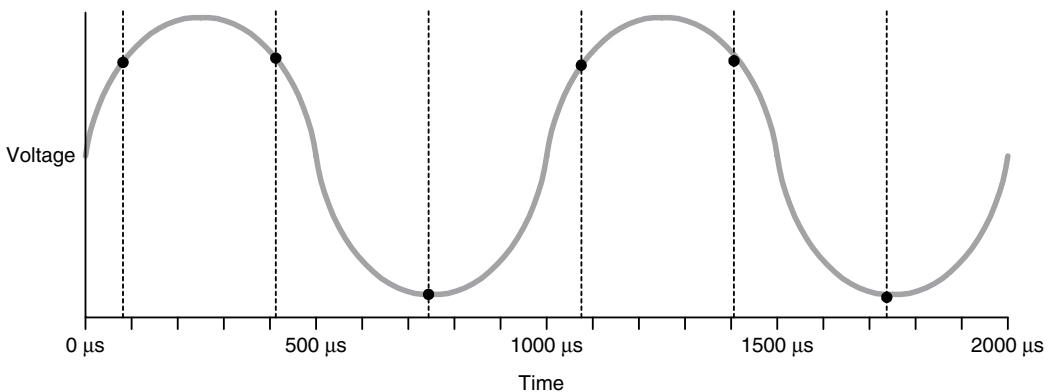


FIGURE 15.5 1-kHz sine wave sampled at 3 kHz.